I-6. RAYLEIGH DISTANCE AS A NORMALIZING RANGE FOR BEAM POWER TRANSMISSION

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Rayleigh distance is the axial distance from a radiating aperture to a point at which the path difference between the axial ray and an edge ray is $\lambda/4$ (Figure 1). To a good approximation, $R_r = D^2/2\lambda$.

Rayleigh also calculated the efficiency of power transmission from a circular lens to a circular focal area, obtaining the well-known formula

$$\eta = 1 - [J_o(u)]^2 - [J_1(u)]^2$$

where

$u = \pi r/F\lambda$

r = radius at focus

$F = f/D$

f = focal length

D = lens diameter

λ = wavelength.

Assume that the diameter of the focal area is equal to the diameter of the lens (a condition discussed by Rayleigh), then

$$u = \frac{\pi D^2}{2f\lambda} = \pi \frac{R_r}{f} = \frac{\pi}{q}$$

where q is the focal length normalized to the Rayleigh distance, or $q = f/R_r$.

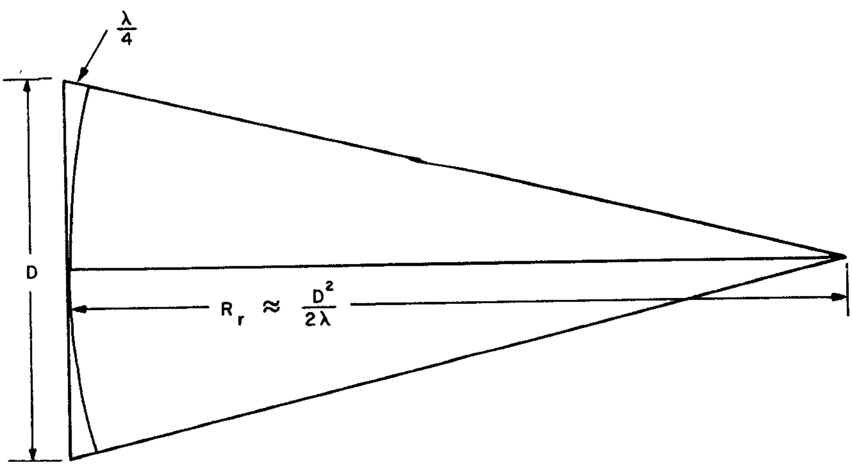

Figure 1. Rayleigh Distance Determined by Proximity Phase Error

Now $r = D/2$ corresponds to power transmission from an antenna to a matched load of the same shape and area, representing "parallel beam" transmission. The normalized range (q) is a dominant parameter in this situation.

Consider the case where the receiving load captures the Airy disk. Then, the first zero of the focal pattern $J_1(u)/u$ is $u = 1.22\pi$ giving $q = 0.82$, and the efficiency of transmission is 84 percent, an often-quoted result.

If the aperture had been square, the focal spot would have been square and the focal pattern (of sin u/u type) has null lines at $u = \pi$ in the principal planes. Then $u = \pi$ and $q = 1$; that is, at the Rayleigh distance, the receiving square area equal to the transmitting square area captures the focal lobe. This case is canonical and is illustrated in Figure 2 with the corresponding case for the circular aperture.

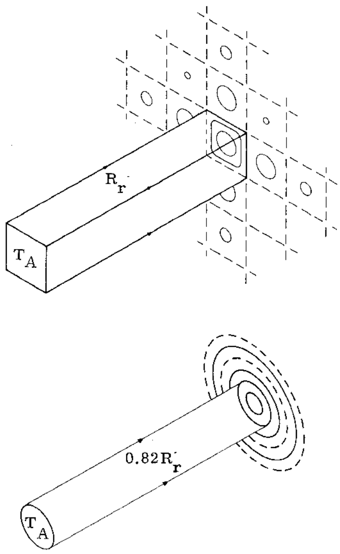

Figure 2. Transmission Along the Rayleigh Range with Square and Circular Beams

The ranges for main-lobe capture with the illumination tapered are shown in Figure 3. The concept of main-lobe capture is heuristic rather than optimum and has the merit of simplicity.

For illumination tapers of the circular transmitting aperture of the inverted parabola family

$$F(\rho) = (1 - \rho^2)^p$$

the transmission efficiency is

$$\eta = 1 - \left[\Lambda_p(u)\right]^2 - \left[\Lambda_p'(u)\right]^2$$

where the Lambda function is defined by

$$\Lambda_p(u) = p!\ J_p(u)/(u/2)^p$$

and the prime denotes differentiation. This formula reduces to Rayleigh's for $p = 0$.

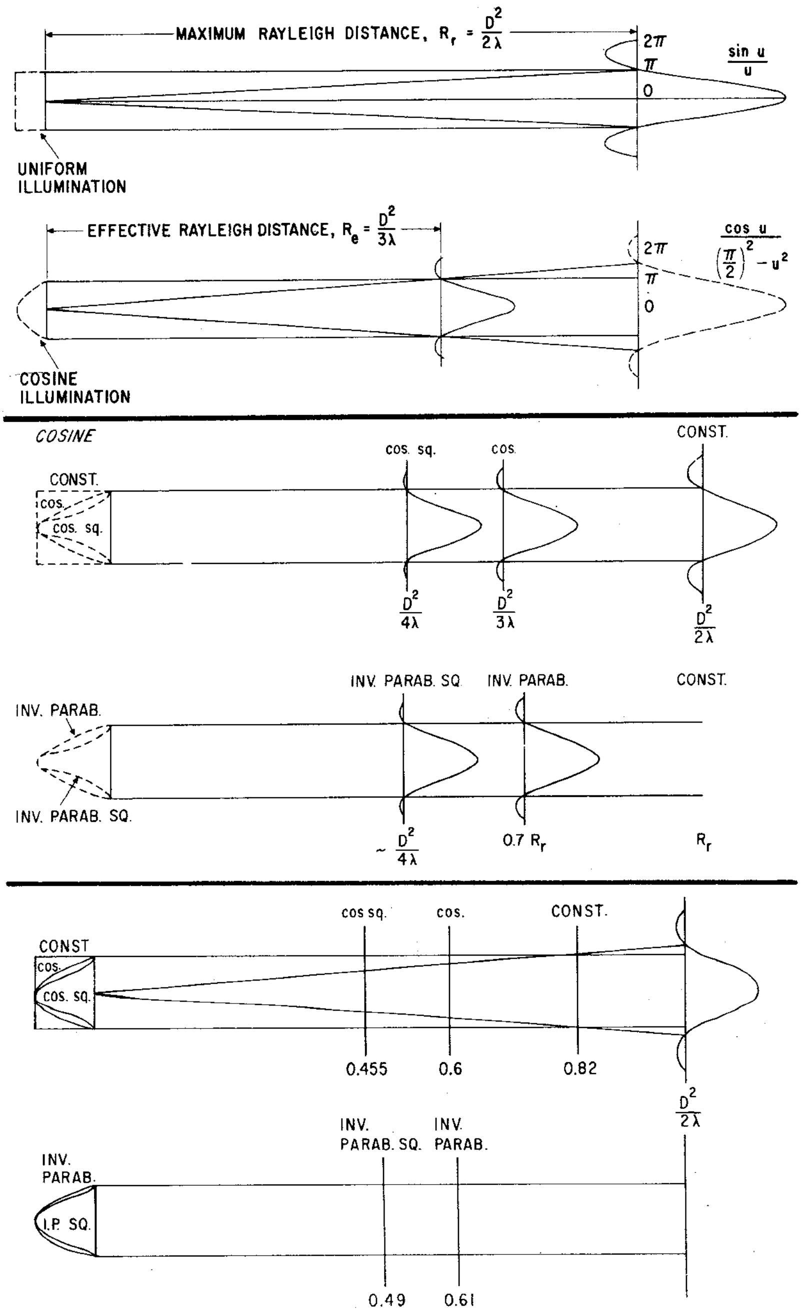

Figure 3. Effective Range Dependence on Illumination Taper at Transmitter

The curves of Figure 4 give the efficiency as a function of u for p = 0, 1, 2, and can be used for transmission between equal or unequal apertures. For equal apertures, the curves relate the efficiency to the normalized range.

For p = 2 and q = 0.49 (approximately one-half the Rayleigh distance), η = 99.66 percent which represents a transmission loss of 0.015 db. The modern beam waveguide using Gaussian-type illumination has a periodic spacing of the order of q = 0.5 with theoretical loss per iteration of 0.002 db.

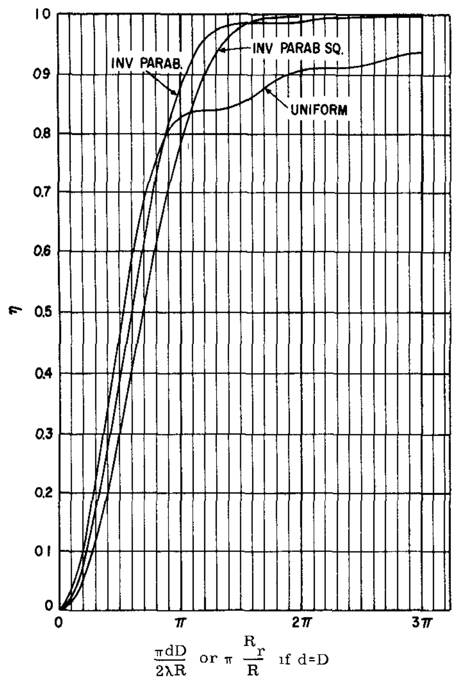

Figure 4. Transmission Efficiency as a Function of System Parameters and Illumination Taper

Transmission between unequal apertures requires the larger aperture to be focused on the smaller (Figure 5). The focal length or range is then less than the Rayleigh distance. For a fixed-aperture ratio, the range also depends on the illumination function of the transmitter.

The Rayleigh distance ($D^2/2\lambda$) also enters into the analytical functions of Fresnel diffraction. For the square aperture, Fresnel integrals or Lommel functions of order 1/2, 3/2 apply. For the circular aperture with equiphase uniform illumination, a transverse power pattern at range R is given by

$$P(w,u) = \left[U_1(w,u)\right]^2 + \left[U_2(w\ u)\right]^2$$

where $U_n(w,u)$ is the Lommel function of two variables ($w = \pi/q$ and $u = \pi d/qD$ where $q = R/R_r$). Thus, the field description depends explicitly only on the ratio of the apertures and the range length normalized to the Rayleigh distance. If the transmitter is focused, its focal length becomes an additional variable that can also be normalized to the Rayleigh distance.

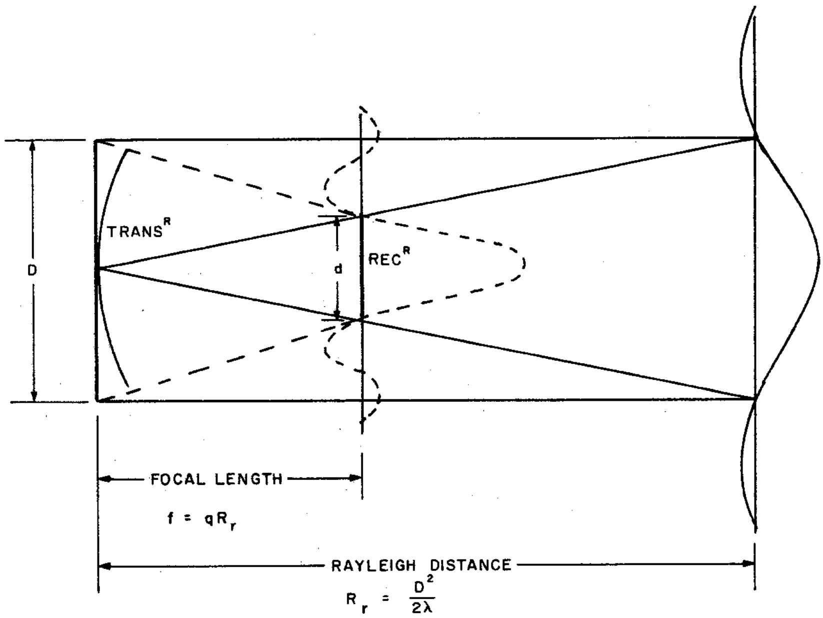

Figure 5. Unequal Square Apertures (Transmitter Focused Within Rayleigh Distance)

The axial power densities of square or circular apertures (Figure 6) can be expressed in terms of the normalized range. For uniform illumination, these are, respectively:

$$P_s = 4 P_a \left[C^2(1/\sqrt{q}) + S^2(1/\sqrt{q})\right]^2$$

$$P_c = 4 P_a \sin^2 (\pi/4q)$$

where

P_a = the aperture power density

$q = R/R_r$

C, S = Fresnel integrals.

Similar formulas can be obtained for tapered and focused illuminations.

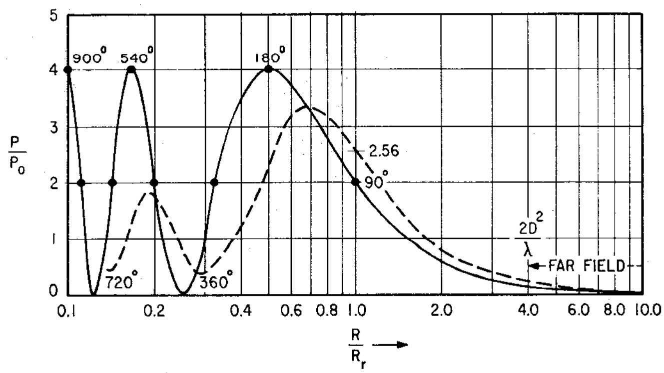

Figure 6. Axial Power Densities

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